Real-Life Applications of Sine and Cosine Functions

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Rationale

The Common Core State Standards – Mathematics (CCSSM) require that students know how to persevere in problem solving.[i] This standard works in conjunction with the content standards. I believe my students often learn the content objectives and lack the ability to persevere when confronted with problems they consider difficult or of little interest. My students often struggle with understanding how the mathematics learned in class is utilized in the world around them. My intent is to use the concepts of trigonometric functions that my students are learning in class and add application problems centered on these concepts to deepen student understanding of how sine and cosine functions are utilized and modeled within the world around them. The intent is to further their knowledge of trigonometric functions and enable them to understand that these mathematical concepts are applicable to their world in many different avenues and scenarios. I intend to present more open-ended scenarios that will require deeper thought than a homework question from our text – problems that provoke students to consider the how and why of the concepts and deepen understanding beyond a mere plugging of numbers into an equation in the proper places. I am hoping that students will engage in the problems and persevere in finding how each scenario leads to deeper content knowledge.

Sine and cosine functions can be used to model many real-life scenarios – radio waves, tides, musical tones, electrical currents. When I consider how to address the Precalculus objectives "to solve real-life problems involving harmonic motion"[ii] and "use sine and cosine functions to model real-life data,"[iii] I struggle with finding problem solving scenarios that require my students to persevere. The introductory lessons in my textbook seem to lack the necessary building of understanding required to enable students to delve into problem solving scenarios. The application problems presented in our textbook are very procedural and typically only require students to understand the various parts of these functions enough to plug values into an equation. These typical textbook problems are not comparable to the more open-ended problems and what is required of students on the Delaware pre and post-tests for Precalculus. The goal of my unit is to present problems that require students to move beyond simple applications of these concepts toward wrestling with how and why these concepts are applicable to a situation – problem solving scenarios that will effectively offer my students an opportunity to persevere in ascertaining and then applying the concepts of trigonometric functions in a "real-life" scenario.

Student population

The Precalculus classes at Conrad Schools of Science are diverse in both age and ability. My students are in grades ten through twelve, have chosen this honors math class with or without meeting stated math prerequisites, come from urban and suburban backgrounds, and represent ability levels from mediocre to gifted, scoring from a two to a four on the Delaware Comprehensive Assessment System. Students have completed the Algebra 2 curriculum in either eighth grade – for my tenth graders – or the previous year for the eleventh and twelfth grade students. Trigonometric functions have been introduced, but the concepts of period, amplitude, scaling, and translations of trigonometric functions have not been solidified. Precalculus students are required to understand how sine and cosine functions model the real world. This requires that students understand how the graphs/functions change according to the given scenario. My students need to understand different aspects of these functions – period, amplitude, and phase shift – in order to know how to apply that understanding to different situations and scenarios.

My students are very social. They enjoy working in groups and being able to ask their peers for possible methods of problem solving. If I can present scenarios that engage students and allow them to conquer problems together, I believe all my students can benefit from the lesson. Utilizing peer sharing and cooperative learning opportunities allow struggling students an opportunity to ask/receive assistance as well as engage all students in mathematical communication. I believe most students are more successful in persevering on a given problem if they have others with which to share their thoughts and ideas.

Following a block schedule allows me to have my students for ninety minutes every other day. This class length will let me offer problem solving scenarios that take a little more time for the typical classroom activity. I intend to utilize scenarios that students can complete and then share with their classmates in presentation type style, whether a poster, PowerPoint, or other creative media. This should encourage my students to persevere in the problem solving as well since they will be presenting their solutions to their peers. I intend to teach this unit in the early spring of the 2013/2014 school year.

Unit objectives

- Students will be able to use definitions and points on the unit circle to evaluate trigonometric functions for $-2\pi \leq t \leq 2\pi$.
- Students will be able to recognize the shapes and key points of sine and cosine graphs.
- Students will be able to find the amplitude and vertical shifts of a transformed sine or cosine function by finding its vertical stretch factor and center between maximum and minimum values.

- Students will be able to find the period and phase shift of a transformed function $f(x)=a\ sin(bx\text{-}c)+d$ or $f(x)a\ cos(bx\text{-}c)+d$ by finding the one-cycle interval given by x-values satisfying $bx\text{-}c = 0$ and $bx\text{-}c = 2\pi$.
- Students will be able to model and solve problems involving simple harmonic motion by relating characteristics of a harmonic motion experiment to the basic characteristics of sine and cosine functions.
- Students will be able to analyze how amplitude, frequency, and tension influence changes in the wave motion of sinusoidal functions.

Historical Background

My students' knowledge of trigonometric functions began in their Geometry course as they studied circles and chords. I make this statement based on the research I found as I started to study the history of the mathematics involved in my unit. Chords and circles are at the beginning of sine and cosine functions. I always considered the teaching of trigonometry to begin with plane triangles. However, as I researched trigonometric functions, I found that the earliest uses – dating to the early ages of Egypt and Babylon – were related to chords of a circle with a fixed radius and how the length of the chord of the various angles (x) around the unit circle were 2sin(x/2).[iv] The roots of trigonometry are founded in astronomy, and trigonometry was used to calculate the positions of stars and planets. I think my students will be as amazed as I was to discover that the Greek astronomer and mathematician Hipparchus in 140 BC produced a table of chords and had methods for solving spherical triangles. His work – based on astronomical observations – contributed to further developments of trigonometry concepts by Menelaus and then Ptolemy – who divided the circle into 360 parts and calculated chords of regular polygons.[v]

The discovery of sine tables throughout history illustrate how the concepts from the Greeks and their chord table inspired the Indian – or Hindu – table of Sines.[vi] These mathematicians constructed their table not just for right triangles in a semicircle but for right triangles in the first quadrant followed by constructing a table of Sines for any angle. The use of history or historical references has never been a method of instruction I utilize in my mathematics classes. However, I find it significant to discover that brilliant scholars – including Euclid, Ptolemy, and Archimedes – wrote works that could be compiled and applied to the study of trigonometric identities establishing the relationship between the six fundamental trigonometric functions, sums and differences, and offering more efficient astronomical calculations.[vii] I think my students will be interested to know that the sine and cosine functions were developed in an astronomical context while tangent and cotangent came from the study of shadows. Teaching the history behind these functions and illustrating how we still utilize similar scenarios should spark a deeper interest in the mathematical concepts for those students who prefer history to math.

Teaching my students the relationship between degrees and radians can now be deepened as I encourage them to see how an arc on the circle with the same length as the radius of that circle creates an angle of one radian. They can easily see how a full circle corresponds to an angle of 2π radians. Seeing how the formula for the circumference of a circle relates to the angle of 2π radians should enable students to comprehend radians to a deeper degree. Roger Cotes (1714) is generally credited with the concept of radian measure even though other mathematicians, including Euler, were measuring angles using arc lengths.[viii] The naturalness of radian measure when using angular measures, the simple derivative calculations in radian measure, and the lack of units make the use of radians in physics and other mathematics using trigonometric functions not only easier but simpler to understand.

When I taught Geometry last year, I knew that right triangles were an introduction to the trigonometric functions I teach in my Precalculus class. I had my students using similar triangles to calculate the height of the flagpole from measures of the yardstick and flagpole shadows. In my research, I discovered that the “shadow stick” is an ancient device found in early civilizations and was utilized to observe the Sun’s motion and tell time similar to the device on a sundial.[ix] Now when I show a clip from the television series Num3ers to my Precalculus students, I can share how the method of using spherical calculations in the tenth century is similar to how Charlie, the mathematician in the show, uses shadows and GPS to find a criminal and solve the crime. Drawing parallels to ancient mathematicians may be a way to engage reluctant learners in mathematical applications of today. I intend to incorporate the historical significance of mathematical concepts into current practices and applications so my students can compare and contrast the relevance of mathematics to people of multiple civilizations past and present.

Mathematical Content

Students begin their study of trigonometry in Precalculus with an introduction to radian measure and the definitions of trigonometric functions on the unit circle. My students know the definitions of vertex and angle, but many have not seen the angle on the coordinate plane nor defined initial side, terminal side, or standard position of the angle (See Figure 1). The first section of the trigonometry chapter starts with these definitions and illustrations and then places angles on the unit circle and introduces radian measure and arc length. Students calculate and sketch coterminal angles and review complementary and supplementary angles in terms of π.

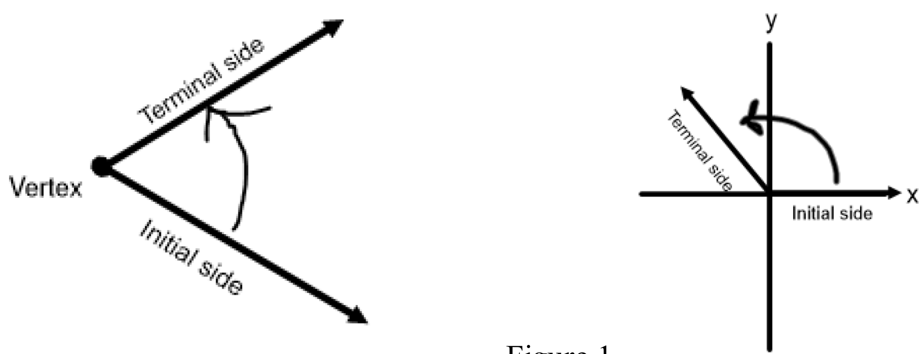
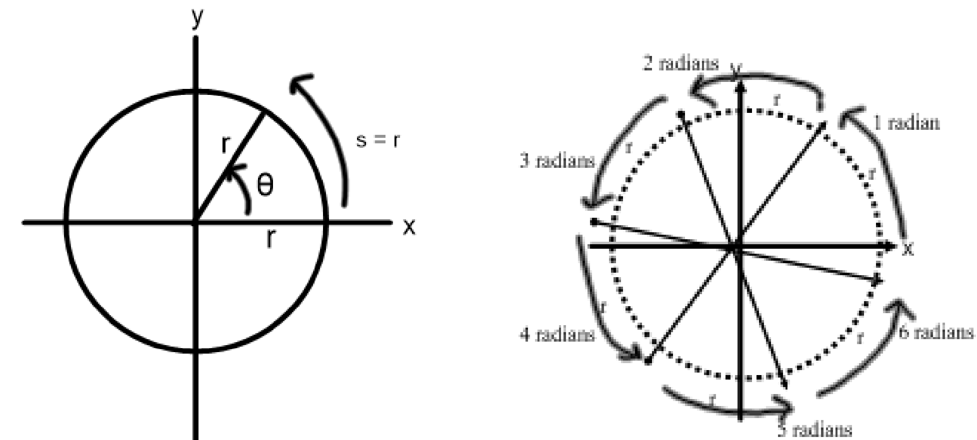


Figure 1

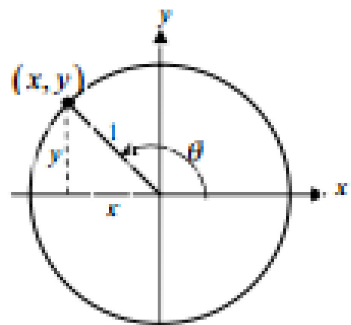


Arc length = radius when θ = 1 radian

Figure 2

Converting between degree and radian measure is then addressed. Students learn the definition of one radian as the measure of a central angle θ that intercepts an arc *s* equal in length to the radius *r* of the circle. Algebraically: $\theta = \frac{s}{r}$. Students use their prior knowledge of the circumference of a circle, C = 2π, to find that a central angle of one full revolution (counterclockwise) corresponds to the arc length s = 2πr and use that information to obtain the radian measure of common angles like ½, ¼, and 1/6 of a revolution (See Figure 2). The ratios 360° = 2π radians and 180° = π radians is used to obtain 1 radian = 180°/π and 1° = π/180°. These ratios enable students to quickly calculate between degree and radian measures. Students finish the first section of the chapter finding arc length and applying the formula for the length of a circular arc to analyze the motion of a particle moving at a constant speed along a circular path. They calculate line and angular speed of various scenarios to understand how and when it is useful to measure how fast a particle moves and how fast the angle is changing.

Students then consider the unit circle and imagine a real number line wrapping around the circle in a counterclockwise direction for positive numbers and a clockwise direction for negative numbers. This activity illustrates how each real number is represented by a point (x, y) on the unit circle by adding or subtracting integer multiples of 2π. The six trigonometric functions are then defined on the unit circle corresponding to the value of each angle measure, θ (See Figure 3). The intent is for students to understand the relationship of sine of the angle equaling the y-coordinate and cosine of the angle equaling the x-coordinate.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Figure 3

In past years, I have not required my students to memorize the exact values of the trigonometric functions for the common angle measures of 30°, 45°, 60°, etc. around the unit circle. However, I intend to do a paper folding activity that should solidify these values without the need for memorization. Students fold 30, 45, and 60 degree angles and measure their values. Students see the relationship between 30 and 60 degree angles and this experience strengthens their understanding of the relationship between the cosine of 30 degrees and the sine of 60 degrees. Students find that for any ordered pair on the unit circle (x,y) : cos $\theta = x$ and sin $\theta = y$. Once students realize that the values around the circle of common angles only differ by the sign – positive or negative according to the quadrant – their understanding deepens and students quickly learn the values of these common angles (See Figure 4).

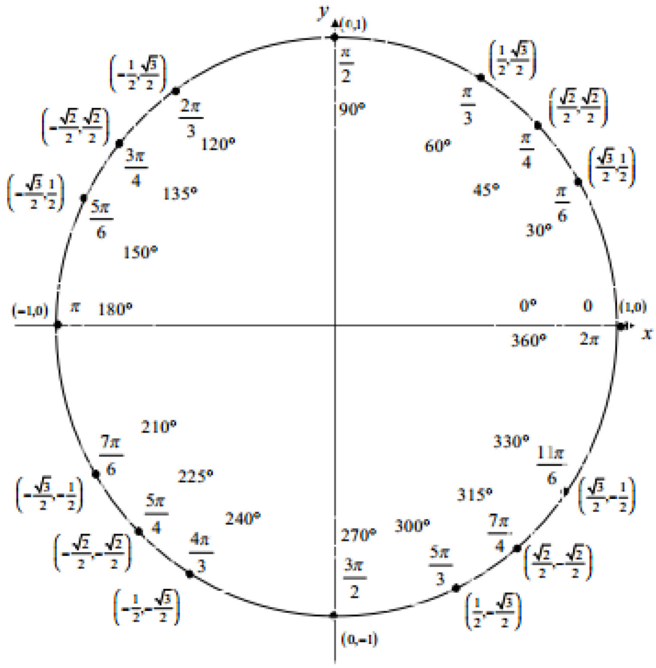
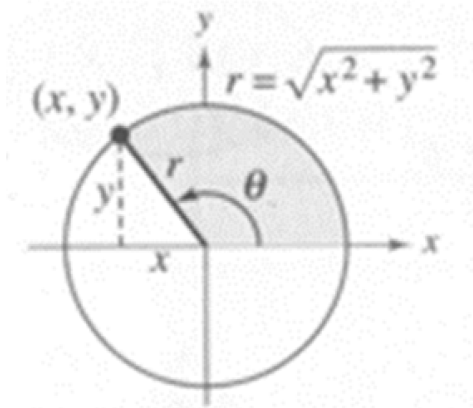


Figure 4

Students then move into studying trigonometric functions from the perspective of right triangles. This section seems to be the easiest for students as they have a strong background knowledge of right triangle trigonometry. They use the mnemonic soh-cah-toa to problem solve. From these concepts, we move into using trigonometric identities: reciprocal identities, quotient identities, and Pythagorean identities. Students are expected to use these identities to problem solve and make real world applications involving right triangles. Students enjoy these applications as they are very familiar with using right triangles whether on the flagpole, a skateboard ramp, or measuring across a distance. The text adds direction and bearing into the problem solving scenarios and my students engage with these problems as if they were studying to be pilots or ship's captains. They solve problems involving depths of submarines, angles of towers, lengths of zip lines, angles for ski slopes, pitches on baseball fields, directional bearings for pilots, lengths of routes for cruise ships, and a sundry of other real world applications for right angle trigonometry. My students grasp these concepts and willingly problem solve the various scenarios given to them in the text or outside sources.

Circular function definitions, where θ is any angle



$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

Figure 5

Moving into the study of trigonometric functions for *any* angle begins to confuse some of my students. They now are given a third set of formulas for sine of an angle (See Figure 5) and struggle with making the connection between these three sets of formulas and how these formulas are essentially all the same. They struggle with remembering that the only difference is they are no longer on the unit circle where the radius is equal to 1 unit. Even knowing the Pythagorean Theorem and teaching that the radius of the circle is equal to $r = \sqrt{x^2 + y^2}$ where $r \neq 0$, students struggle with evaluating these six functions. Then we use reference angles and reference triangles to extend their understanding of non-acute angles, but I feel that my students struggle with this section of the chapter. I may need to strengthen their understanding of the unit circle by helping students understand that the y-coordinate gives you the sine and the x-coordinate gives you the cosine before I utilize any pendulum, Ferris wheel, or harmonic motion activities that introduce students to sine and cosine curves.

When I begin teaching graphs of sine and cosine, the next section in the chapter, I discover that my students are familiar with the shape of these graphs. However, the definitions of amplitude, period, and scaling are not as familiar. My students quickly understand how the amplitude, the absolute value of one half the distance between the maximum and minimum values of the function, is determined. They also quickly grasp and understand how the relationship between scaling by vertical stretching and shrinking is connected to these concepts covered in their study of quadratics. Students clearly see how vertical stretches and shrinks affect the amplitude. However, the concepts of period, one complete cycle of the curve, and phase shift, horizontal translations of the curve, are not easily understood by my students. They can see the differences on the graphs but struggle when asked to put these changes into an equation. They attempt to memorize that in the sine equation of $y = a \sin bx$ and cosine equation $y = a \cos bx$, that the period = $2\pi/b$ when b is a positive number. And they write in their notes that the left and right endpoints of a one cycle interval is determined by solving the equation $bx - c = 0$ and bx

– c = 2π. Yet on an assessment, they struggle with making those calculations. It is very frustrating as these concepts are the very concepts they struggle with on the state pre and post tests for Precalculus.

My text offers students a one page section on simple harmonic motion using a ball on the end of a string. When assuming ideal conditions of no air resistance or friction and perfect elasticity, the ball will move in a uniform and regular pattern. I believe students need to be exposed to more than this one example in order to understand how motion of this nature is described by the sine or cosine function. My intent with this unit is to address these struggles with multiple applications in real world scenarios in the hope that working through multiple scenarios will facilitate student understanding of these concepts. Using the Delaware tidal charts to plot points, tidal charts of other areas, and multiple Ferris wheel problems, I am hopeful that these scenarios will engage students in greater understanding and knowledge of sine and cosine functions. These should help students see how the graph changes and enable them to interpret how the changes on the graph are reflected in changes to the equation. I decided to utilize multiple scenarios of Ferris wheels as that seems to be the chosen scenario on the various assessments my students are required to complete. I am hopeful that my students will not only persevere in the problem solving of real world applications, but they will build their understanding of how these scenarios enable them to understand the concepts and prepare them for success on required assessments.

Classroom Activities

I am choosing to begin the activities for this unit with a unit circle activity that requires the students to fold circles to the 30, 45, and 60 degree angle measures and glue them to a white background unit circle. See Activity 1. Then they will use a tracing paper wheel to mimic rotations around the unit circle as they investigate the coordinates of common angles around the circle. See Activity 2. Graphing tidal charts – Activity 3 and 4 – and investigating families of Ferris wheels – Activity 5 – should enable students to solidify the concepts of sine and cosine functions in real world scenarios.

Activity 1: Unit Circle Activity[x]

Objective

Students will gain an understanding of the angles and coordinates around the unit circle using a color coding system to represent common reference angles.

Materials per student

- 1 copy of each circle – blue, green, yellow, white
- Scissors

- Glue/glue stick
- Protractor
- Brads, tracing paper, and graph paper for Activity 2 The Rotating Wheel

Directions

1. Cut out the blue, green and yellow circles. Do NOT cut the white circle.

2. On the white circle, draw your x-axis and y-axis. Label your coordinate points. Remember your radius on the unit circle is 1. Add your angle measurements for these 4 points.

3. Using the BLUE circle: Fold the circle in half twice. Hold the piece with the two folds on the left and the single fold on the bottom. With your protractor in the corner of the pie piece, draw a 45° angle. Hold the corner of the pie piece and cut along the line you just drew (cut slightly above the corner, not through it). Once you reach the outside of the circle, cut down to the single fold, forming a 45-45-90 right triangle. This right triangle has a hypotenuse of 1 (because that's the radius of the circle). Calculate the side lengths of this 45-45-90 triangle ($\sqrt{2}/2$ and $\sqrt{2}/2$). Label your lengths. Unfold the triangle (so it looks like a bow tie) and glue it to the white circle with the triangle you just labeled in quadrant I. Next, label your angles around the unit circle and use your side lengths to write the coordinates of each angle (45, 135, 225, 315 degree angles).

4. Using the GREEN circle: Follow the same instructions except you are measuring a 60° angle with your protractor from your center corner creating a 30-60-90 triangle. With a hypotenuse of 1, calculate and label your side lengths (1/2 and $\sqrt{3}/2$). Glue your "bow tie" on top on the blue "bow tie" with the 60° angle at the origin. Using reference angles, write the angles and coordinates of these points on your circle.

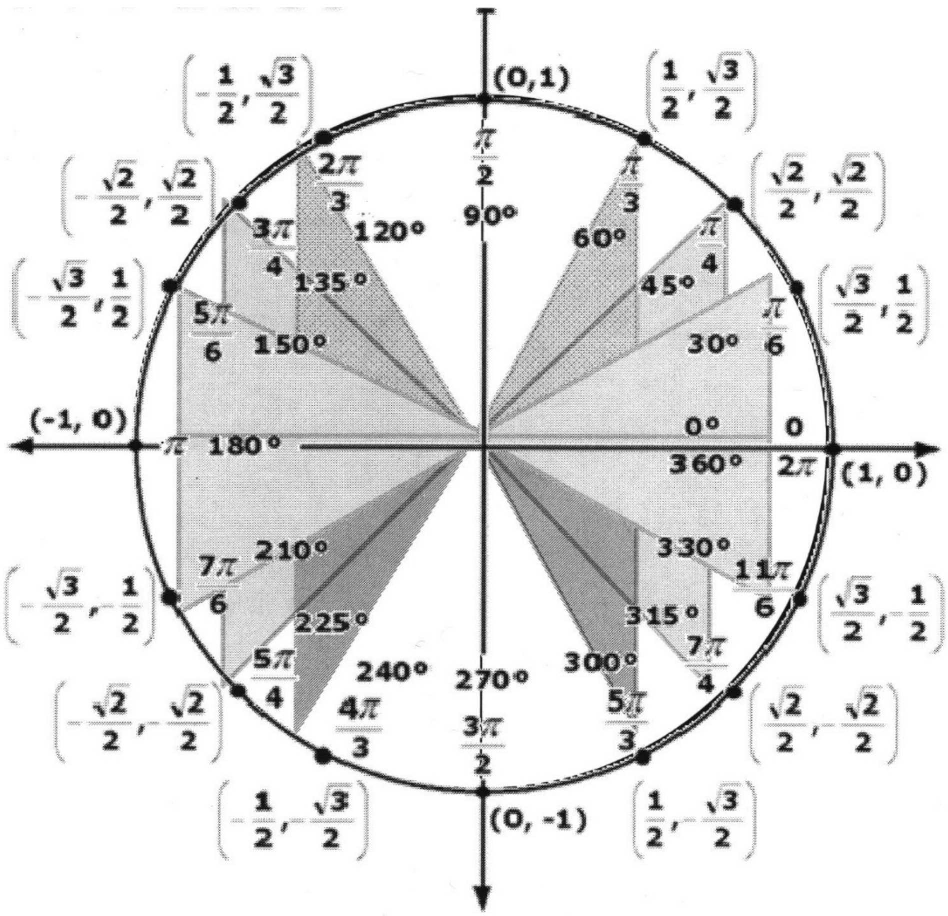
5. Using the YELLOW circle: Follow the same instructions except you are measuring a 30° angle from the corner of your pie creating a 30-60-90 triangle. Label your side lengths (1/2 and $\sqrt{3}/2$) and glue your "bow tie" on top of the green one with your 30° angle at the origin. Using reference angles, write the angles and coordinates of these points on your circle.

6. Your finished product should have these common angles measured in degrees and radians as well as the coordinates of each of these points.

7. Using a sheet of tracing paper, draw a circle the same size as your white circle. Draw a line from the center of the circle right to the circle's edge along the x-axis.

Put a point and label that point as Point A. This circle will be used as your Ferris wheel in upcoming investigations.

Your Finished Product – Figure 6



Activity 2: Coordinates of Points on a Rotating Wheel

Objective

Students will use their unit circle from Activity 1 and the tracing paper wheel to discover how the sine and cosine functions track the position of a point on the circumference of a circular object as the object rotates around a center point.

Directions

Using a brad fastener, attach your tracing paper circle to your unit circle through the origin with the tracing paper wheel on top. Align your Point A at the point (1, 0). Follow the directions below and answer each question as you complete the activity.

1. As you turn your tracing paper wheel in a counter-clockwise direction:

   a. How does the x-coordinate of Point A change?

   b. How does the y-coordinate of Point A change?

   c. How are the patterns of change in coordinates extended as the wheel continues to turn counterclockwise through more revolutions?

2. Find and record angles of rotation between 0° to 360° that will take Point A to the following special points:

   a. Maximum and minimum distance from the horizontal axis

   b. Maximum and minimum distance from the vertical axis

   c. Points with equal x- and y- coordinates

   d. Points with opposite x- and y- coordinates

3. Rotate the wheel to an angle of 70°. Point A, that started at (1, 0), is now located at A' (0.34, 0.94). Explain how the symmetry of the unit circle allows you to deduce the location of Point A after rotations of 110°, 250°, and 290°.

4. Using graph paper, mark along the x-axis the angle measures 0, 45, 90, 135, 180, 225, 270, 315, and 360 degrees.

   a. Plot your y coordinates for each of these angles. Using a graphing calculator – in degree mode – graph the equation f(x) = sin x. Adjust your window and compare your graph to the graph on the calculator. In particular, do the graphs show the same maximum points, minimum points, and intercepts? Using smooth curves, draw the sine curve on your graph. Below your angle measures, record the radian measures for each angle.

   b. Now plot your x coordinates for each of these angles on the y-axis. Using your graphing calculator, graph the equation g(x) = cos x. Do the graphs show the same maximum points, minimum points, and intercepts? Using smooth curves, draw the cosine curve on your graph.

5. How will the x coordinate and y coordinate of Point A change during a second complete revolution? How will those patterns be represented in graphs of the coordinate functions for $360° \leq \Theta \leq 720°$?

6. Using your knowledge of similar triangles, if your radius was a value greater than 1, how would the measures around your unit circle change? Think about the values on each axis first and then the values between the x and y axes.

7. Application: At the Delaware State Fair, there is a Ferris wheel with a 12 meter radius. Suppose Point A is a seat on the Ferris wheel and the Ferris wheel is positioned on an x-y-coordinate system like your unit circle.

a. You are on a seat to the right of the vertical axis halfway to the highest point of the ride. Make a sketch showing the wheel, x- and y- axes through the center of the wheel, and your starting position.

b. Find the x- and y- coordinates of your seat after the wheel has rotated counterclockwise through an angle of 65°. Through an angle of 155°.

c. Write expressions that give your position relative to the x- and y- axes for any counterclockwise rotation of Θ in degrees from your starting point.

d. Suppose the wheel turns at one revolution per minute. Describe in several different ways the location of your seat at the end of 50 seconds.

Activity 3: Delaware Tides and Sinusoidal Functions

Objective

Students will graph tidal data and see how this periodic phenomena yields a sinusoidal graph of a cosine function.

Materials

Philadelphia Tidal Data
Graph paper

Task

1. Using the Tidal Data in the chart, graph the Tide height on the y-axis and Time on the x-axis.

2. Does the data resemble a sine or cosine curve? How do you know?

3. What are the period, maximum height, minimum height, and vertical displacement of the water?

4. Use your graph to predict the height of the water in 3 hours? 6 hours? 12 hours?

Philadelphia (USCG Station), Delaware River, Pennsylvania Tidal Data

December 12, 2013
39.9333° N, 75.1417° W

Date	Time	Height (ft)
2013-12-12	12:01 AM EST	3.437369
2013-12-12	01:01 AM EST	2.310932
2013-12-12	02:01 AM EST	1.024503
2013-12-12	03:01 AM EST	-0.205341
2013-12-12	04:01 AM EST	-0.439499
2013-12-12	05:01 AM EST	0.679190
2013-12-12	06:01 AM EST	2.200295
2013-12-12	07:01 AM EST	3.596456
2013-12-12	08:01 AM EST	5.049646
2013-12-12	09:01 AM EST	6.047442
2013-12-12	10:01 AM EST	6.029712
2013-12-12	11:01 AM EST	5.353620
2013-12-12	12:01 PM EST	4.444671
2013-12-12	01:01 PM EST	3.379062
2013-12-12	02:01 PM EST	2.118284
2013-12-12	03:01 PM EST	0.654278
2013-12-12	04:01 PM EST	-0.316026
2013-12-12	05:01 PM EST	0.084470
2013-12-12	06:01 PM EST	1.397469
2013-12-12	07:01 PM EST	2.730622
2013-12-12	08:01 PM EST	4.127709
2013-12-12	09:01 PM EST	5.405495

2013-12-12   10:01 PM EST   5.779031
2013-12-12   11:01 PM EST   5.232115

Activity 4: Tidal Charts and Cosine Functions[xi]

Objective

Students will collect, organize, and analyze real-world data of high and low ocean tides to calculate a function to model the data or behavior of the tides.

Materials

NOAA Tides and Currents website or
Tidal charts from other areas of the world
Graph paper
Graphing calculators

Rationale

Students will discover tidal behavior in other parts of the world and use the data to create graphs, write equations, and make predictions based on their data.

Task

You are the harbor master of your Oceanside town. People coming into and out of your harbor need to know in advance about the behavior of the tides near your town. Tides are caused by the pull of the sun and moon on the oceans and upon the rotation of the earth. However, the exact tidal pattern at a coastal location is also strongly influenced by the shape of the coastline and the profile of the nearby sea floor. By recording the height of the tides in your location over a period of time, you will be able to use your measurements and predict future tides. Remember that the fishing vessels and boat tours depend upon your ability to predict tidal behavior.

1. Choose a region of the world where ocean tides are measured.

   My location is ____________________________________.

2. Collect three days of data for your region gathering data for 2 high tides and 2 low tides per day. Record this information in a t-chart using time for the independent variable and tide height for the dependent variable. Begin with time 0 on day 1 and end with time 72 on day 3.
3. Using your data from your chart, graph your data connecting the points with a smooth curve and scaling the axes according to your needs.

4. Analyze your data: Follow these directions to find the equation of the sinusoid that best fits your data. Use the general form y = a cos (bx – c) + d.

   a. Find "d", the vertical shift by taking the average of the average of your high tides and the average of your low tides (the average of the averages).

      Enter your "d" value here ___________________

   b. Find "a" the amplitude. The amplitude is the distance between your average height and "d" (or your average low and "d"). Look at your data and determine if you need to make "a" negative.

      Enter your "a" value here ___________________

   c. Enter "b". Use a half lunar day as the period for the tides, 12.4 (12 hours and 24 minutes). Since P = 12.4, b = 2π/12.4.

      Your value of "b" is <u>2π/12.4 or 5π/31</u>

   d. Find "c" the horizontal shift. Let c equal the time of your first tide.

      Enter your value of "c" here ________________

   e. Put it all together and write the equation of the cosine curve in the form from y = a cos (b(x-c)) + d.

      Enter your equation here __________________

   f. Enter your equation in your calculator and using the table function, graph your equation on the same graph as your original data. What observations can you make about the two graphs? How does your equation compare with your tidal data?

Activity 5: A Family of Ferris Wheels

Objective: Students will use sine and cosine functions to model Ferris wheel scenarios in order to deepen their understanding of the graphs of these functions.

Rationale: This activity allows students to investigate how changes in amplitude and period are manifested in the equations and graphs of sine and cosine curves. Tracking the location of seats on a spinning Ferris wheel shows how the sine and cosine functions can be used to describe rotation of circular objects. Students will discover how variations in the basic functions are useful in modeling a variety of periodic patterns.

Materials:
Handout
Graphing calculators or software that allows students to change parameters and view corresponding graphs

Task: You are the owner of the Soaring Round the Heights Ferris wheel company. You have decided to present to customers a family of Ferris wheels that will appeal to all riders – children through adults. Family 1: You are to make a presentation based upon given specifications that will show customers how each Ferris wheel will offer them a fun ride – include the maximum and minimum heights on the ride as well as the time of one revolution on the wheel. Family 2: You must use the measurements of your Ferris wheels to make a presentation that clearly explains to customers their height above the ground of each of your rides and the time of one revolution. Remember that Ferris wheels turn in a counterclockwise direction.

Ferris Wheel Family 1: Use the equations to find the maximum and minimum heights on the ride, the amplitude, the period (the time it takes for one revolution), and the radius of the wheel.

1. You have attached a light to one of the seats on your Ferris wheel. The height of the light is given by $h(t) = 20 – 19\sin(120)t$ where height is in meters and t is in minutes. Label this Ferris wheel 1-1.

2. Ferris wheel 1-2 is modeled by the equation $h(t) = 15 \sin 0.5t + 17$.

3. Ferris wheel 1-3 is modeled by the equation $h(t) = -12 \cos t + 14$.

Ferris Wheel Family 2: Use the given measurements to make a presentation clearly describing each scenario. Include an equation and graph for each Ferris Wheel.

1. The first Ferris wheel has a radius of 10 meters and the bottom of the wheel is 2 meters about the ground. The wheel rotates at a constant speed, and takes 100 seconds to complete one revolution. Write the equation that models this Ferris wheel, graph your equation and label it as Ferris wheel 2-1.

2. Ferris wheel 2-2 has a radius of 20 meters and travels at a rate of 5 revolutions per minute. Riders board at the bottom chair from a platform 1 meter above the ground.

3. The third Ferris wheel in this family could be one for young children. This wheel has a radius of 15 feet and is 2 feet off the ground. The wheel spins at 1 revolution per 2 minutes. Clearly show why children would not be afraid to ride this wheel.

4. The last Ferris wheel in this family has a radius of 30 meters, the center is 35 meters above the ground, and each revolution takes 10 minutes.

Conclusion

As my students finish the above classroom activities, it is my hope that their understanding of sine and cosine functions will be strengthened beyond the depth of previous years of teaching these concepts. Implementing these activities will afford students a deeper understanding of the manipulations of these functions as well as offer them a solid understanding of real life scenarios of sine and cosine functions.

Appendix A – Common Core State Standards

The following Common Core State Standards represent a list of standards addressed in this unit.

- CCSS.Math.Content.HSF-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- CCSS.Math.Content.HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- CCSS.Math.Content.HSF-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$,$f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- CCSS.9-12.F.1F.6 Extend the domain of trigonometric functions using the unit circle.
- CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- CCSS.Math.Content.HSF-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the

unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
- CCSS.Math.Content.HSF-TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- CCSS.9-12.F.TF.6&7 Model periodic phenomena with trigonometric functions.
- CCSS.9-12.F.1F.7 Analyze functions using different representations.
- CCSS.Math.Content.HSF-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
- CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP4 Model with mathematics.

Bibliography

Brown, Stephen I., and Marion I. Walter. *The art of problem posing*. 3rd ed. Mahwah, N.J.: Lawrence Erlbaum, 2005.

Fey, James Taylor. *Core-plus mathematics: contemporary mathematics in context.*. 2nd ed. New York, N.Y.: Glencoe/McGraw-Hill, 2009.

Flater, David. "WWW Tide and Current Predictor." Tide/Current Predictor. http://tbone.biol.sc.edu/tide/tideshow.cgi (accessed November 14, 2013).

Gersbach, Amy, and Ingrid Williams. "NCTM Conferences." precalculus.shawnee.site.eboard.com. http://precalculus.lrhsd.org (accessed November 7, 2013).

"History of Trigonometry." History of Trigonometry. http://www.trigonometry-help.net/history-of-trigonometry.php (accessed November 7, 2013).

Joyce, David. "History of Trigonometry Outline." History of Trigonometry Outline. http://www.clarku.edu/~djoyce/ma105/trighist.html (accessed October 28, 2013).

Larson, Ron, Robert P. Hostetler, Bruce H. Edwards, and David C. Falvo. *Precalculus with limits a graphing approach*. 5th ed. Boston, Mass.: Houghton Mifflin, 2008.

Common Core State Standards Initiative. http://www.corestandards.org/Math (accessed October 28, 2013).

O'Connor, J J, and E F Robertson. "The trigonometric functions." Trigonometric functions. http://www-history.mcs.stand.ac.uk/HistTopics/Trigonometric_functions.html (accessed November 5, 2013).

O'Connor, JJ. "Roger Cotes." Cotes biography. http://www-history.mcs.st-andrews.ac.uk/Biographies/Cotes.html (accessed November 2, 2013).

Polya, George. *How to solve it; a new aspect of mathematical method.*. Princeton, N.J.: Princeton University Press, 1945.

Rogers, Leo. "The History of Trigonometry- Part 1." : nrich.maths.org. http://nrich.maths.org/6843 (accessed November 1, 2013).

Rogers, Leo. "History of Trigonometry - Part 3." : nrich.maths.org. http://nrich.maths.org/6908 (accessed November 2, 2013).

Schoen, Harold L.. *Teaching mathematics through problem solving: grades 6-12*. Reston, VA: National Council of Teachers of Mathematics, 2003.

Smoller, Laura. "The birth of trigonometry." The birth of trigonometry. http://ualr.edu/lasmoller/trig.html (accessed November 2, 2013).

Curriculum Unit Title: Real-Life Applications of Sine and Cosine Functions

Author: Karen Brown

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will define the six trigonometric functions from a right triangle perspective and as functions of real numbers. They will use both perspectives to graph trigonometric problems ascertaining how changes affect the graph and equation. Students will model periodic phenomena as they solve application problems involving sine and cosine functions.

ESSENTIAL QUESTION(S) for the UNIT

How are the sine and cosine functions defined for radian measure or real numbers? How are the patterns of change extended in the coordinates around the unit circle? How can trigonometry be used to model circular motion? When modeling periodic patterns of change, how do changes in the scenario affect the equation $f(x) = a \sin bx + c$ and $f(x) = a \cos bx + t$ for values of a, b, and c?

CONCEPT A	CONCEPT B	CONCEPT C
Trigonometric Functions: on the Unit Circle and of any Angle	Graphs of Basic Sine and Cosine Functions	Model periodic phenomena with trigonometric functions

ESSENTIAL QUESTIONS A	ESSENTIAL QUESTIONS B	ESSENTIAL QUESTIONS C
How do you evaluate trigonometric functions by using the unit circle? How do you evaluate trigonometric functions for any angle?	How do you sketch the graphs of sine and cosine functions? How are changes in the period and amplitude reflected on the graph?	How do you use trigonometric functions to solve real life problems? How are changes in the scenarios reflected in the equation and in the graph?

VOCABULARY A	VOCABULARY B	VOCABULARY C
Radian Unit circle Definitions of the 6 trigonometric functions Reference angle Revolution/ Frequency	Sine curve Cosine curve Period Phase shift Amplitude Intercepts	Translations Trigonometric Model

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Materials: colored paper, scissors, glue, brads, protractors, tracing paper, graph paper, tidal data, graphing calculators, unit handouts

End notes

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[i] "Common Core State Standards for Mathematics," accessed October 28, 2013, http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf, 6.

[ii] Larson, Ron et al., *Precalculus with limits a graphing approach,* 331.

[iii] Larson, Ron et al., *Precalculus with limits a graphing approach,* 277.

[iv] Smoller, Laura, "The birth of trigonometry." The birth of trigonometry.

[v] Joyce, David. "History of Trigonometry Outline." History of Trigonometry Outline.

[vi] O'Connor, J J et al., "The trigonometric functions." Trigonometric functions.

[vii] Rogers, Leo. "History of Trigonometry - Part 3." http://nrich.maths.org/6908 accessed November 2, 2013.

[viii] O'Connor, JJ. "Roger Cotes." http://www-history.mcs.st-andrews.ac.uk/Biographies/Cotes.html accessed November 2, 2013.

[ix] Rogers, Leo. "History of Trigonometry – Part 1." http://nrich.maths.org/6843 accessed November 1, 2013.

[x] Gersbach, Amy et al. "NCTM Conferences." precalculus.shawnee.site.eboard.com. accessed November 7, 2013.

[xi] Ibid.

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Students will define the six trigonometric functions from a right triangle perspective and as functions of real numbers. They will use both perspectives to graph trigonometric problems ascertaining how changes affect the graph and equation. Students will model periodic phenomena as they solve application problems involving sine and cosine functions.

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CONCEPT A	CONCEPT B	CONCEPT C
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ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
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VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
Radian Unit circle Definitions of the 6 trigonometric functions Reference angle Revolution/ Frequency	Sine curve Cosine curve Period Phase shift Amplitude Intercepts Maximum points Minimum points	Translations Trigonometric Model Simple Harmonic Motion

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Materials: colored paper, scissors, glue, brads, protractors, tracing paper, graph paper, tidal data, graphing calculators, unit handouts