Where Is The Congruency In Your World?

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Introduction

The concept of congruence is the topic of this unit written as part of the Delaware Teacher Institute Geometry Seminar. I teach Enrichment Math and Language Arts at Jennie Smith Elementary School. Jennie Smith is a Title I school with a population of approximately 500 Kindergarten through fifth grade students. My unit is written for the fifth grade students I service in Enrichment Math. My class size is usually capped at 20 but historically my fifth grade class has had 12-17 students in attendance.

The Enrichment program is a pullout program servicing first through fifth graders. The students I see are classified as high achievers, capable of working with more advanced content than is introduced in the regular classroom setting. Often the students I serve are capable of working with content one to two grade levels above their current grade, and sometimes even higher. I've selected 'congruence' as my topic for this Geometry unit because I believe it provides an opportunity for students to delve deeper into geometry, working with angles, triangles, and other polygons at a comprehensive level. It also introduces advanced vocabulary, which, based on my experience with previous classes, students love to be exposed to. I am 100% sure that my students are not familiar with the accurate use of the terms proposition, theorem, and proof as it relates to math. I want my Enrichment Math students to use this vocabulary fluently and effortlessly as they talk about geometry and congruence.

Rationale

Often students who achieve at a higher performance level do not demonstrate adequate academic growth throughout the school year because they are often not exposed to content beyond what they already know or their current grade level. I expect this unit to satisfy their thirsting minds for challenge beyond what they would normally be exposed to in their regular classroom setting. While I plan to challenge my fifth grade Math Enrichment students, I recognize that there will be a need to differentiate the content based on learning styles and/or readiness levels. Not all high achieving students learn in the same way and not all come with the same background knowledge. For these reasons I plan to include learning opportunities that will enhance all learning preferences of the visual; kinesthetic; auditory; verbal, both oral and written; interpersonal; as well as the intrapersonal learners through offering choices for demonstrating what they have learned.

Objectives

This unit will provide students with a relatively comprehensive understanding of congruence as it relates to, knowing what congruence is, discovering how it is proved, analyzing two (and some three) dimensional shapes to determine if they are congruent, disputing whether or not two (or more) shapes are congruent, providing reasons and supporting evidence as to why or why not, and evaluating the relevance and importance of congruence. My intent is for my students to be able to apply what they've learned about congruence to real world situations through authentic problem solving. I expect students to be able to identify the appropriate tools necessary and to use them appropriately in their proofs of congruence.

The types of authentic applications I anticipate as being components of my unit are its relevance to art, architecture, and construction. My intent is for this unit to be flexible enough to include additional authentic applications of congruence that may surface from discussions with my fifth grade students as I teach this unit.

With regard to art, one thing that comes to my mind is line designs that originate from a single point and extend to different points spaced equidistantly from each other. One thing that has intrigued me about line designs is the resulting curve made by the intersecting lines. It would be interesting for me, and I assume for my students as well, to be able to explain this concept as it may relate to congruence or find out that it does not and be able to present and support the findings. At this point I am not sure whether the concept of congruence applies to this phenomena but I am interested in investigating it further to determine if it does so that I can direct my students' learning.

As I looked around my classroom, rooms in my home, and spaces in commercial buildings I thought about whether or not the idea/concept of congruence is conscientiously addressed in architecture and construction. It appeared to me that in order for specific structures to be straight and/or symmetrical there had to be some level of congruence with some of the angles. These are some of the types of questions I want my students to address, investigate, and ponder. They are certainly not at a disadvantage of structures to look.

The Vocabulary

To facilitate a better understanding of this unit I thought it best to start with basic vocabulary to aid the reader. As I conducted my research and read articles on congruence I continued to come across mathematical terms that were not readily accessible in my memory banks. So, to relieve you of the task of trying to discern the meaning of this terminology I've decided to include the definitions, explanations, and examples, where appropriate, of Euclid's Elements here, including proposition, theorem, proof, axiom, and postulate. Believe me, it will make your life so much easier as you read the mathematical foundation for congruence according to Euclid's Elements.

So, what exactly is Euclid's Elements? "Euclid's Elements is a composite of logical mathematical statements that explain geometry and arithmetic. It was written by the Greek scholar, Euclid, to prove that what was seen was true and how it was known to be true in the 4^{th} Century BCE. It is considered to be the first textbook on Geometry, has been translated throughout the world, and serves as a model for clear logical reasoning". This explanation of Euclid's Elements was derived from information provided on TheMathPage.com. [Plane Geometry, Intro to Euclid's Elements]

A *proposition* is defined as a statement or sentence that is either true or false based on the information provided within the context of the statement. Examples of a proposition are 'Henry Brown is a nurse', 'Eight minus three is five', 'Seventeen plus six is forty', 'Francis is six feet tall', 'The sum of an even number plus an odd number is an even number', and 'Three times four is twelve'. As you can see from the examples provided some of the statements are true, some are false, and some can be determined to be true or false. It is necessary to determine if in fact the proposition is true.

As defined in the Math Dictionary of ICoachMath.com, a <u>*theorem*</u> is a statement that has to be proven. After a theorem has been proven it can then be used to prove other statements. A proposition that has been determined to be true is a theorem.

As presented in the Mathematical Proof page of Wikipedia, a <u>*proof*</u> is an illustration or demonstration of the validity of a statement. It must show that the statement is valid in all situations without exception. Proofs can be formal as well as informal.

According to the Math dictionary found at ICoachMath.com, an <u>*axiom*</u> is a rule or statement accepted to be true without a proof. An example of an axiom would be the identity property of addition, which states that the sum of any number plus zero is equal to the given number.

A <u>*postulate*</u> can be defined as an assumption that serves as the basis for mathematical reasoning. It establishes the existence or truth of a basic idea. A postulate is accepted as a true statement and does not need to be proved. It can be considered to be a synonym for axiom.

It should be noted that the websites cited above were last accessed November 14, 2011.

Euclid's Elements

The foundation for this unit on Congruence is structured around specific definitions, postulates, and propositions from Euclid's Elements. To fully grasp the concept of Congruence some aspects of the additional concepts of Symmetry, and Transformations are incorporated as they relate to Congruence. The concept of Similarity is also introduced in this unit. The definitions, postulates, common notions, and propositions relevant to this unit, taken from Book 1 of Euclid's Elements, as presented in the interactive version of Euclid's Elements by ED Joyce at Clark University, follow. It should be noted that some of the principles are presented verbatim as stated in Joyce's translation of Euclid's Elements. I found the illustrations that were on this website to be helpful and I recommend that teachers review this website prior to teaching this unit. From a historical perspective, it provides the opportunity to see the logic of Geometry in action. Much, if not all of what is presented via the definitions, postulates, and propositions, can be proven or demonstrated via construction or drawing. To construct the proofs accurately a straight edge and a compass must be used. It is important to discourage, or better disallow, your students from constructing any proofs freehand.

Definitions

Some of the definitions in Euclid's Elements are merely discussions of concepts while others are descriptions of new concepts based on terms previously defined. Students should be encouraged to think and speak like a mathematician when discussing math concepts. The definitions included in this unit will aid in promoting appropriate use of mathematic language during student discussions. To help students follow and visualize the definitions as stated, drawing with a straight edge and paper folding are the best tools. Folding and cutting or tearing apart a Tangram puzzle from a single square piece of paper is a perfect example of a folding activity that will help to facilitate the understanding and recognition of congruence. Here are some examples of definitions that could lead students to rich, meaningful, productive, and eye opening discussions.

Definition 10 – When a straight line standing on a straight line makes the measures of the adjacent angles equal to one another, each of the angles is a right angle with a measure of 90 degrees. The straight line standing on the other is considered to be perpendicular to the line upon which it stands. Students can demonstrate an example of this definition by drawing an illustration.

Definition 11 – An obtuse angle is an angle whose measure is greater than the measure of a right angle or greater than 90 degrees.

Definition 12 – An acute angle is an angle whose measure is less than the measure of a right angle or less than 90 degrees.

Optional tasks students can perform that relate to Definitions 11 and 12 include identifying obtuse angles and acute angles that are part of structures in the classroom, that are on pictures that you provide from photographs, magazines, or other publications, or from a set of triangular manipulatives that you may have in your classroom.

Definition 13 – A boundary is that which is an extremity or border of anything.

Definition 14 – A figure is that which is contained by any boundary or boundaries.

An activity that students can participate in to comprehend the terms defined in Definitions 13 and 14 is to label the boundary of given shapes and pictures of objects and label or describe the corresponding figure that results from the given boundary. Additionally, you may want to have students color the boundary in a specific color and the resulting figure in a different color. A real-world example of this concept could be a garden with a fence around it. These two terms will be addressed later in an activity in Lesson Ones.

Definition 19 – Rectilinear figures are figures contained by straight lines. Trilateral figures are contained within three straight lines. Quadrilateral figures are those contained by four straight lines, and multilateral figures are contained within more than four straight lines. Students can demonstrate examples of the different figures that are part of this definition by drawing illustrations that correspond to the number of straight lines. Here is another opportunity to have students label and use the terms boundary and figure from Definitions 13 and 14 of this definition.

Definition 20 – With regard to trilateral figures, an equilateral triangle is a triangle that has its three sides equal in length, an isosceles triangle is a triangle that has only two of its sides equal in length, and a scalene triangle has three sides whose lengths are unequal. This definition classifies triangles by their symmetry. An equilateral triangle has three lines of symmetry, an isosceles triangle has one line of symmetry, and a scalene triangle has no lines of symmetry.

Definition 21 – With further regard to trilateral figures, a right-angled triangle has a right angle as one of its angles. An obtuse-angled triangle has an obtuse angle as one of its angles, and an acute-angled triangle has three acute angles. This definition classifies triangles by the measure of the angles they contain.

A good activity to engage students in for Definitions 20 and 21 is to provide them with a variety of trilateral figures, commonly referred to as triangles, and have them sort them on a mat that is divided into three columns with the headings Equilateral Triangles, Isosceles Triangles, and Scalene Triangles. For the purpose of enhancing student vocabulary you may want to replace the word ‘Triangle’ with ‘Trilateral’ in the headings and discuss the difference between ‘trilateral’ meaning 3 sides and ‘triangle’ meaning 3 angles. After students have sorted the trilateral figures/triangles, have them explain the results of their sorting by justifying their selections with information from the definitions.

Definition 22 – With regard to quadrilateral figures, a square is both equilateral and right-angled. An oblong or rectangle is right-angled but not equilateral. A rhombus is equilateral but not right-angled. A rhomboid or parallelogram has its opposite side lengths and angle measures equal to one another but is neither equilateral nor right-angled. Quadrilaterals other than those mentioned are called ‘trapezia’ or trapezoids. A fun interactive activity for students to engage in for this definition is for them to state whether a figure that you display on the overhead projector, interactive white board, computer screen, or other means is a square, a rhombus, rhomboid/parallelogram, or trapezoid. It is a good idea to have a physical version of the figure as well so that students can prove how they chose to identify the figure.

Postulates

Each Postulate in Euclid’s Elements is an axiom that is accepted as true without proof that is specific to the subject matter of plane geometry. Here are some Postulates that are good for a fifth grade discussion.

Postulate 1 – To draw a straight line from any point to any point.

Postulate 4 – That all right angles equal one another.

Common Notions

Common notions are axioms that refer to various kinds of magnitudes or numerical values.

Common Notion 1 – Things that are equal to the same thing are equal to each other. An example to illustrate this notion is: if A is equal to C and B is equal to C then A is equal to B. This represents the property of transitivity of the equality relation

Common Notion 2 – If equals are added to other equals then the resulting wholes are also equal. An example to illustrate this notion is: if A is equal to B and C is equal to D then A + C = E and B + D = F results in E and F being equal. This represents the concept of simplification.

Common Notion 3 - If equals are subtracted from other equals then the resulting remainders are also equal. An example to illustrate this notion is: if A is equal to B and C is equal to D then A - C = E and B - D = F results in E and F being equal.

Common Notion 4 – Things that coincide with one another equal one another. This is interpreted to mean that if something can be moved or placed on something else so that they coincide with each other they are equal to each other. This common notion is relevant to Proposition 4 identified below.

Propositions

The following Propositions are important results in the theory of congruence. Some give us sufficient conditions for two triangles to be congruent and others give us tools for the exploration of properties of other figures.

Proposition 4 – If two triangles have lengths of two sides equal to lengths of two sides respectively, and have the measures of the angles that are contained by those straight lines equal, then the length of bases are also equal, the triangle is identical to the triangle and the measures of the remaining angles equal the remaining angles respectively, specifically those opposite the equal sides. This represents the SAS (Side-Angle-Side) principle of triangle congruency.

Proposition 8 – If two triangles have two sides equal to two sides respectively and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines. This represents the SSS (Side-Side-Side) principle for triangle congruency.

Proposition 13 – If a straight line stands on a straight line then it makes either two right angles or angles whose sum equals two right angles. This proposition is used in the proof of Proposition 15.

Proposition 15 – If two straight lines cut or intersect one another, then they make the measures of the vertical angles equal to one another.

Proposition 26 – If two triangles have two angles whose measures equal two angles respectively and one side length equal to one side, specifically either the side adjoining the equal angles or the side opposite one of the equal angles then the remaining side lengths equal the remaining sides and the remaining angle measure equals the remaining angle. This represents the ASA (Angle-Side-Angle) principle of triangle congruency.

Congruence

Congruence is defined as having the same shape and size. It is important that students learn and use the precise terminology in geometry. The definition of Congruence says that two triangles are congruent if the lengths of all corresponding sides are equal and the measures of all corresponding angles are equal. Usually, students conclude congruence of two shapes if they can flip (reflect), slide (translate), or turn (rotate), or any combination of these transformations, one of the shapes so that it can be superposed exactly onto the other.

Students may become confused when identifying congruent shapes or figures if they are looking at two figures that differ by one characteristic, specifically size. If that happens, it is important to introduce the concept of Similarity and to make the distinction between congruency and similarity. With regard to Similarity, it is important to reinforce that two shapes are similar if the only difference between the two is size. The formal definition of similarity of two triangles says that they need to have the corresponding angles of the same measure and the corresponding sides need to be proportional by the same factor. As in congruency, there are propositions that give us necessary conditions for two triangles to be similar as well (AA, SAS, SSS).

To prepare students to think in terms of comparing sizes of corresponding angles and sides, first we should make sure that students understand congruence of line segments and congruence of angles. Also, enlarging and shrinking of a segment by a factor is an important step to master before the students compare whole pictures and objects.

Note that the Common Notion 1 can be written with the word “equal” being replaced by the words “similar” and “congruent”. This means that the relations of “being congruent” and “being similar” are transitive, reflexive, and symmetrical relations.

Here is how the Common Notion 1 leads to the following properties for Congruence:

Congruence Property 1: If two triangles are congruent to a third triangle, then they are congruent to each other. For example, if the triangles ABC and EFG are congruent to the triangle LMN, then the Triangles ABC and EFG are congruent to each other.

Congruence Property 2: Any triangle is congruent to itself.

Congruence Property 3: If the triangle ABC is congruent to the triangle EFG, then the triangle EFG is also congruent to the triangle ABC.

Here is how the Common Notion 1 leads to the following properties for Similarity:

Similarity Property 1: If two triangles are similar to a third triangle, then they are similar to each other. For example, if the triangles ABC and EFG are similar to the triangle LMN, then the triangles ABC and EGF are similar to each other.

Similarity Property 2: Any triangle is similar to itself. It’s scale factor is one.

Similarity Property 3: If the triangle ABC is similar to the triangle EFG, then the triangle EFG is also similar to the triangle ABC.

Activities

Prior to starting any of these activities, students should be familiar with how to use a straight edge, a compass, and a protractor. You may want to instruct them separately on the use of these tools and give them ample opportunity to practice with the tools for proficiency of use and safety.

Shapes and figures you will want to have available should include right, obtuse, isosceles, and scalene triangles, regular and irregular pentagons and other polygons, and trapezoids. The larger the variety of polygons you provide for students to explore will result in a richer and broader span of conceptual knowledge that your students will attain.

The concept of congruence is not just reserved for triangles or angles. Inclusion of the various propositions recorded previously exposes my students to the big idea of Congruence and will enable them to relate it to a variety of shapes and figures. They will be expected to identify figures that are congruent and identify the proposition that supports that claim. They will be able and expected to use various methods of proof to claim congruency or negate congruency and explain why. I expect the students to be able to sort shapes that are congruent, visualize, and use the relevant transformation to show congruency through superposition. Students will engage in activities requiring them to identify characteristic properties, the line of symmetry, and the relevant transformations that allowed them to demonstrate and justify congruency. Real world problem solving will be employed in order for students to see and experience the relevance of the idea of congruency in geometry.

Lesson 1 - Identifying and Defining Congruence (Three activities)

Goal

Students will demonstrate their understanding of Congruence by justifying the three Congruence Properties. Students will describe a movement or series of movements that will illustrate why two figures are congruent.

Objective

Students will analyze the concept of congruence through a hands-on and interactive activity. From that exploration they will define congruence in their own words by identifying the characteristics of two congruent figures. This addresses Common Notion 4 as documented in the Background Section of this unit.

Teacher Directions

The teacher should provide sample objects for all figures mentioned in the Definitions and Propositions. Provide students with a vocabulary list of the new terms relevant to this unit. The new geometric terms should be highlighted in the Definitions and the Propositions that the students will use and analyze.

Once students have the shapes fitting properly, one on top of the other, ask probing questions until you receive responses that congruent shapes are the same size and shape respectively. Expand the conversation to what it means mathematically for two figures to have the same shape and same size. You are attempting to reinforce that the measures of the angles are equal and the measure of the side lengths are equal. Make the connection of this language to statements that the angles are congruent and the sides are congruent. It is important to distinguish between the terms ‘equal’ and ‘congruent’. Be sure to emphasize that the term ‘equal’ is used when referring to numbers and ‘congruent’ is used when referring to shapes and/or figures.

Have students discuss the movements made to line the figures up exactly. This will lead into the discussion of transformations that can be used to describe congruence that will be reflected on later in the unit. The set of figures students will use for this activity will include triangles and various polygons. As the unit progresses, students will use the vocabulary list provided to mark the appropriate words as they encounter them and will write the description in their own words. The students will be allowed to refer to their own list and be expected to use the vocabulary correctly every time the word is appropriate in a particular situation or activity discussion. Throughout the process of presenting the definitions, students will be actively involved by drawing using a straight edge, labeling, and sorting through their set of shapes.

To prepare for Activity 1A cut on the dotted lines of the Student Mats for Matching the Congruent Shapes to separate into individual mats, Worksheet 1A-1. Give each student one mat. Put students in groups, preferably in groups of four. Give each group a copy of Worksheet 1A-2. These worksheets can be found in the Appendix.

Activity 1A – Defining Congruence

Have the students cut out, as perfectly as possible, each of the 16 shapes on Worksheet 1A-2 that was given to their group. Have them cut each figure as close to it’s inside boundary as possible. They are to select and compare an appropriate figure with the intent to superpose the figure onto one of the figures drawn on their mat to find a congruent match. For each match, have the students label the congruent pairs, one with a letter and the other with the same letter and the number 1 (example A and A1, B and B1 etc.). They are to continue selecting and comparing the shapes until they have an exact match, the shapes can be superposed one on the other, fitting perfectly on top of each other, for each of the figures on their mats. During the discussion of this activity students should recognize and identify the movements made to accomplish fitting the shapes on top of each other.

Activity 1B Congruence

An online interactive activity to support this lesson is found on the Illuminations website. http://illuminations.nctm.org/ActivityDetail.aspx?id=4 This interactive activity addresses Propositions 4, 8, and 26 documented above. In this activity students are asked to select the component parts that will generate a triangle. The applet will generate the triangle and then another triangle. After comparing the two triangles the students are asked to decide whether the two triangles are congruent or not. They will receive immediate feedback as to whether their response is correct or incorrect. This activity also addresses the three parts of a triangle that have to be the same in order for two triangles to be congruent. This interactive activity provides the opportunity for students to discover what combination of equal parts will not guarantee congruency while they are working with the construction of congruent triangles.

After having grasped the concept, definition, and proof of congruence (same shape and size) by physically fitting the triangles (and other shapes) on top of each other, students will describe the movements that were necessary to superpose the shapes in order to show congruence. Examples include rotating the figure a certain number of degrees, or reflecting it vertically. They will further explore what is different about shapes that are not congruent. From that exploration students will analyze shapes that are similar and then define similarity.

Activity 1C Similarity

Provide students with a real object that they can measure. Have available another version of the same object in a different size. Have the students measure the distances between pairs of points on both figures. This will allow them to see that the ratio of the corresponding distances is the same between the two different figures. A triangle should also be used so that students can measure the angles using a protractor.

Another way that students can visualize and experience similarity is to give them two paper copies of identical quadrilaterals. Have them cut off the same measure, ½ to 1 inch, from one vertical side and one horizontal side from one of the shapes and compare the resulting shape to the original. They should be able to see that the shapes are no longer congruent, but are now similar with the only difference being the size.

Lesson 2 – Drawing and Constructing Congruent Angles (Two activities)

Goal

Students will integrate the concepts and/or ideas from Definition 10, Propositions 13 and 15, and Postulate 4 as the foundation for illustrating congruent angles. After constructing their illustration(s) using the straight edge, students will explain how the illustration is a representation of the relevant Definition and Postulate and proof of the relevant Propositions in their own words demonstrating their own understanding.

Objective

Students will construct congruent angles using a straight edge and compass (optional). Students will explain why the angles they have constructed are congruent relying on the relevant components of Euclid's Elements, presented earlier in the Background section of this unit.

Teacher Directions

Give each student a worksheet with Definition 10, Propositions 13 and 15, and Postulate 4 written at the top of the worksheet. Instruct students to a variety of figures including triangles and quadrilaterals accompanied by one of the propositions or postulate addressed by this activity. For the relevant figures identify points of origin for the lines that students are to draw for the assignment.

Activity 2A – Drawing Congruent Angles

For each of the Propositions addressed in this lesson, have students draw, using the straight edge and optional compass, as many examples as they can to illustrate the concept of the propositions. They should vary the length of the line segments and/or lines. Students should label the angles that are equal to one another. Students should share their drawings either on the overhead projector, if drawn on transparency film, or using a document reader/projector. Ask probing questions that will cause students to determine whether the angles that are equal, according to the propositions, are also in fact congruent. After constructing the congruent angles, student should write a brief explanation that provides adequate information supporting the fact that the angles they have drawn are in fact congruent.

Activity 2B – Construction Congruent Angles

This activity is similar to Activity 2A. The difference is that instead of drawing congruent angles, students will construct the angles using toothpicks and glue. The directions are almost the same with some variation and are included here for ease of access. For each of the Propositions addressed in this lesson, have students construct as many examples as they can to illustrate the concept of the Propositions. They should vary the slope of the toothpicks in order to see how the sizes of the angles vary as well. Students should glue the toothpicks to the worksheet provided and label the angles that are equal to one another. Students should share their constructions either by holding them up and showing them to the class after they've dried or by using a document reader/projector to project them on a screen. Ask probing questions that will cause students to determine whether the angles that are equal, according to the Propositions, are also in fact congruent. After constructing the congruent angles, students should write a brief explanation that provides adequate information supporting the fact that the angles they have constructed are in fact congruent. I recommend using quick drying glue for this activity.

You may choose to use either Activity 2A or 2B but not both or assign each of the activities to different groups of students.

Lesson 3 – Constructing Congruent Figures

Goal

Students will interpret the concepts of Propositions 4, 8, and 26 to construct congruent triangles and prove that they are congruent. Students will be able to explain how the figure(s) represent the proposition using appropriate language.

Objective

Students will construct congruent figures using a Geoboard and rubber bands based on the specific Proposition to model and specific measurement(s) given to them.
After constructing the figures on the Geoboard, students will explain, in their own words, why the figures they have constructed are congruent relying on the proofs posed in Propositions 4, 8, and 26 as the foundation for their explanation.

Teacher Directions

Give each student a Geoboard, a supply of rubber bands, and copies of Propositions 4, 8, and 26 written in kid friendly language. You may choose to pair students with a partner, however each student should still construct his/her own triangles on their own Geoboard. The purpose of pairing students with a partner would primarily be to facilitate pair sharing and discussing their results. Give students a measurement in Geoboard terms, an example of this would be 'side is four peg units long', and the Postulate they will use to construct a triangle that is congruent to the first triangle that they constructed. Giving students the measurement for one side of the triangle only provides the opportunity for discovery of the construction of triangles. Giving them the specific Proposition to model for congruency provides the opportunity for students to figure out different ways the congruent triangle can be positioned on the Geoboard.

Student Directions

Using the Geoboard and rubber bands provided, construct congruent figures that satisfy the criteria specified by the teacher. Construct the first triangle and then based on the Proposition identified as the basis of proof for congruency, construct another triangle that is congruent to the first. Students need to be prepared to explain what the proposition states and how their pair of congruent triangles is an example of that proof.

Geometer's Sketchpad

The next two lessons involve the use of version 5 of Geometer's Sketchpad software. The information provided is taken from the workbook entitled, Exploring Plane and Solid Geometry with the Geometer's Sketchpad version 5 published by Key Curriculum Press. The activities in this workbook are recommended for grade levels sixth through eighth. If you have access to Geometer's Sketchpad version 5 you can explore the lessons identified below and determine if they are appropriate for your students.

Lesson 4 – Triangle Congruence Properties

This lesson activity involves the Three Pairs: Triangle Congruence activity, described in Exploring Plane and Solid Geometry with the Geometer's Sketchpad workbook. This activity provides the students with a sketch that they will use to explore what information is needed to prove that two triangles are congruent. Students are introduced to the symbolic representation of congruence. Some of the discussion topics in this lesson include the six congruence statements that can be made about congruent triangles and whether fewer than six statements can be used to prove that two triangles are congruent. Students are directed to draw figures and name corresponding segments.

Lesson 5 – Transformations

This next lesson emphasizes transformations such as Slides, Turns, and Flips to reflect on the importance and properties of congruence in general. Make sure when you address these lessons that the students know there is a starting position for one of the figures, the first figure, and the outcome of the transformation is the position of the second figure. This activity involves the Slide, Turn and Flip: Exploring Transformations activity from Exploring Plane and Solid Geometry with the Geometer's Sketchpad workbook. It defines a transformation as a change in the size or position of a shape. In this activity students explore the three types of transformations that maintain the size and shape of a figure identified as a translation, which is known as a slide, a rotation often referred to as a turn, and a reflection also called a flip. The terms that are introduced in this lesson are 'pre-image' referring to the original after it has been transformed and 'image' which refers to the resulting figure after the transformation. Students are asked to predict what they think will happen before a transformation and then compare the result.

Culminating Activities

Brainstorming Real-World Relevance

Brainstorm with students why they think the concept of congruence is important. Ask the students to identify specific situations where the absence of congruence could lead to a problem. Some suggestions you might offer to facilitate the brainstorming session include architecture, building construction, art, clothing, carpentry, and shelving. A real world example where the absence of congruence is a problem is the Leaning Tower of Pisa. Suggest that students research other buildings and/or structures that have a problem with congruency and what affect it has had on the building's structure and functionality.

A Simulation of Building from Prefabricated Materials

An activity that would engage our kinesthetic students is to provide them with precut and premeasured pieces of cardboard designed to construct a small building (something like a small bird house). Explain to students that this is actually how some buildings are constructed, with the sections of the building already premeasured and precut needing only to be assembled. As you prepare the pattern pieces be sure to have some angles that do not match up or some rectangular sections that are not exactly straight so that students can discover the gaps that would occur. After students have assembled their structures, engage them in a discussion about the problems that could arise from these gaps and whether or not there are remedies to the situation. Coax them to realize that the gaps occurred because the sections or angles that were supposed to match up with another section were not congruent, as they needed to be.

Annotated Bibliography

The following books served as a point of reference as I developed activities I considered to be appropriate and that offered the amount of breadth I wanted for my students. I list them here for you to refer to in the event that you want more of a variety of activities for your students than what I have presented here.

Tiffany Moore, Geometry: Grades 4-5 (North Carolina: Carson-Dellosa Publishing Company, Inc, 2007) – This book covers the fundamental topics of geometry. It provides a variety of challenging activities for the advanced student as well as practice for those who need the additional support.

Frances M. Thompson, Hands-On Math: Ready-to-Use Games & Activities for Grades 4-8 (New York: The Center for Applied Research in Education, 1994) – This book contains a vast collection of instructional lessons that are relevant to a variety of specific learning objectives. The lessons are designed to include a kinesthetic lesson activity, a visual lesson activity, and an interpersonal lesson activity.

Gary Robert Muschla and Judith A. Muschla, Hands-On Math Projects with Real-Life Applications: Ready-to-Use Lessons and Materials for Grades 6-12 (New York: the Center for Applied Research in Education, 1996) – This book is a collection of hands-on investigations designed to engage students in the application of math

concepts and skills. The projects included encourage problem-solving, critical thinking, and decision-making.

Appendix

Common Core Standards

The following Standards are addressed in this unit either directly or by virtue of the procedures students will engage in to accomplish an activity, assignment, or specific task.

Fifth Grade

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4. Classify two-dimensional figures in a hierarchy based on properties.

Text Types and Purposes

W.5.1. Write an opinion on topics or texts, supporting a point of view with reasons and information. Provide logically ordered reasons that are supported by facts and details. Provide a concluding statement or section related to the opinion presented.

W.5.2. Write informative and/or explanatory texts that examine a topic and convey ideas and information about it clearly. Introduce a topic clearly, provide a general observation and focus, and group related information logically; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. Use precise language and domain-specific vocabulary to inform others about or explain the topic.

Sixth Grade

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Seventh Grade

Draw construct, and describe geometrical figures and describe the relationships between them.

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

The definition of congruent as presented in the Common Core Standards Glossary is: Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

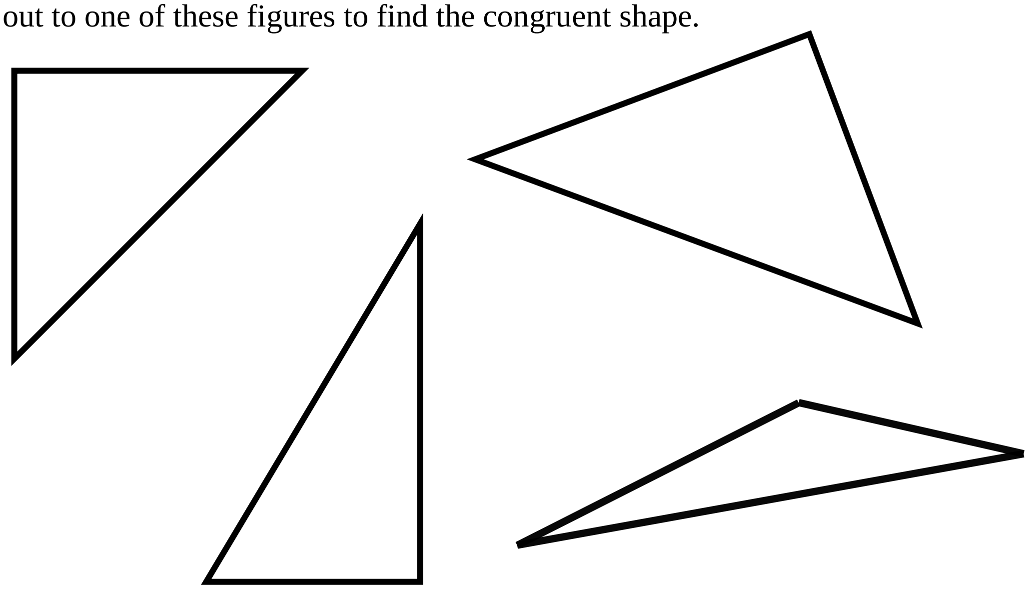
Delaware State Standards

Standard 3 (5–8) – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.

Worksheet 1A-1

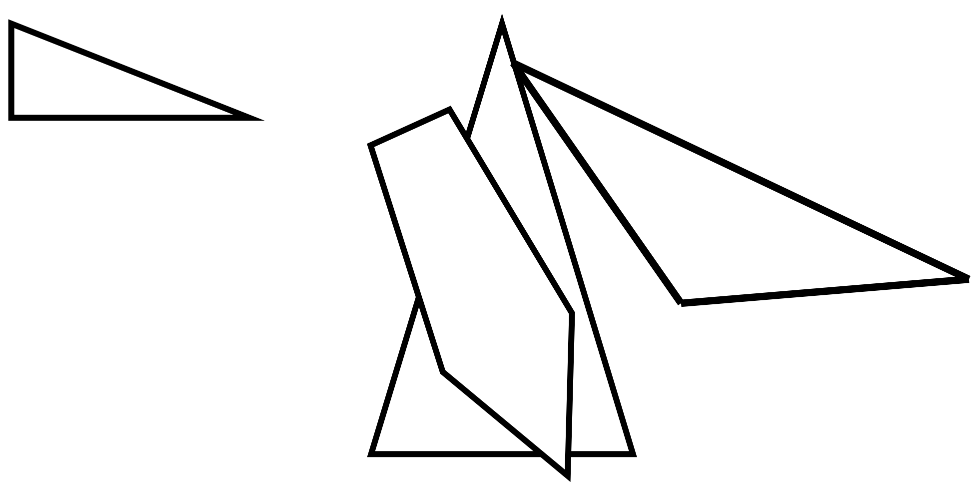
MAT A

Student Mats for Matching the Congruent Shapes. Match a shape from those that were cut out to one of these figures to find the congruent shape.



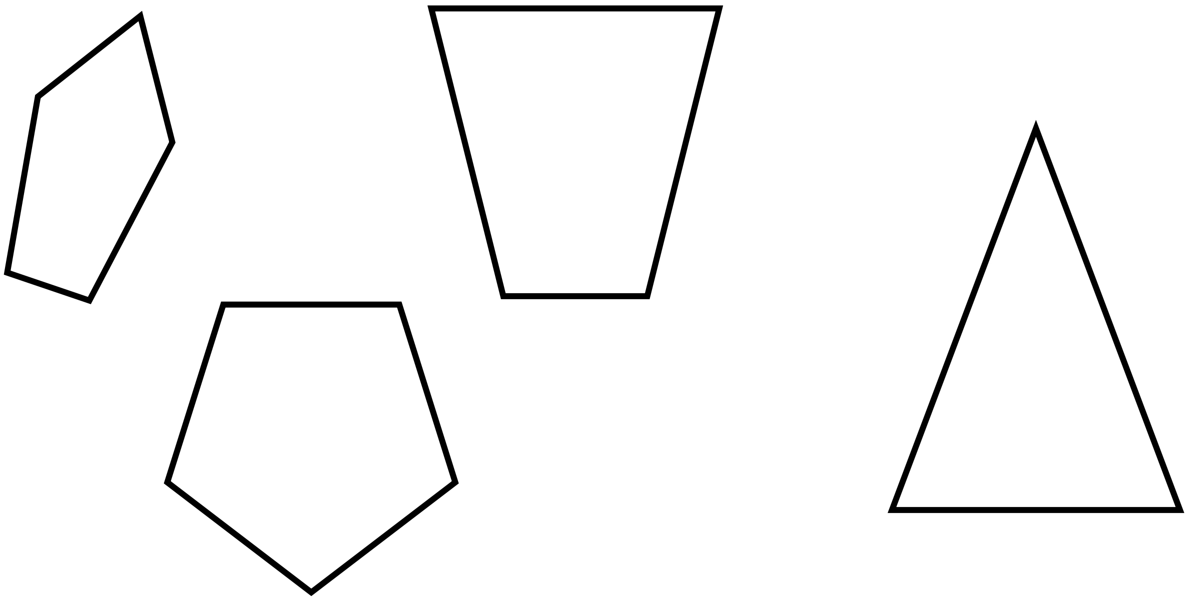
MAT B

Student Mats for Matching the Congruent Shapes. Match a shape from those that were cut out to one of these figures to find the congruent shape.



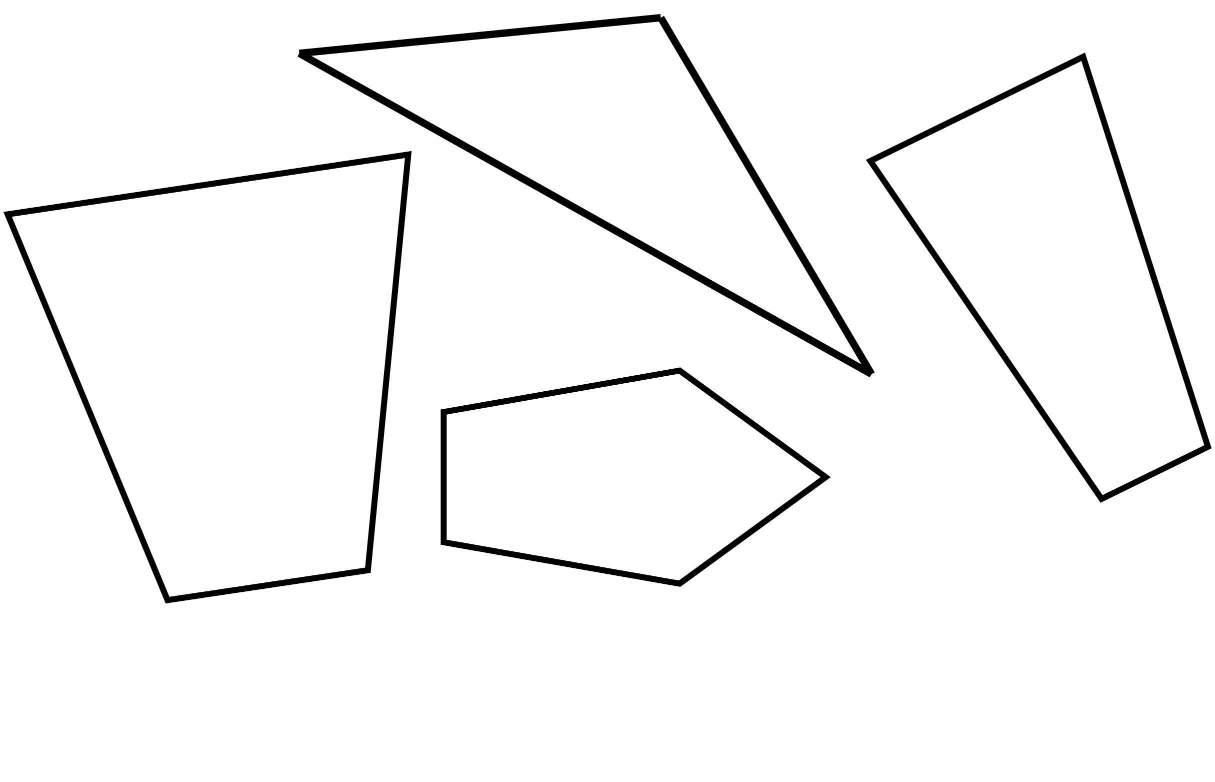
MAT C

Student Mats for Matching the Congruent Shapes. Match a shape from those that were cut out to one of these figures to find the congruent shape.

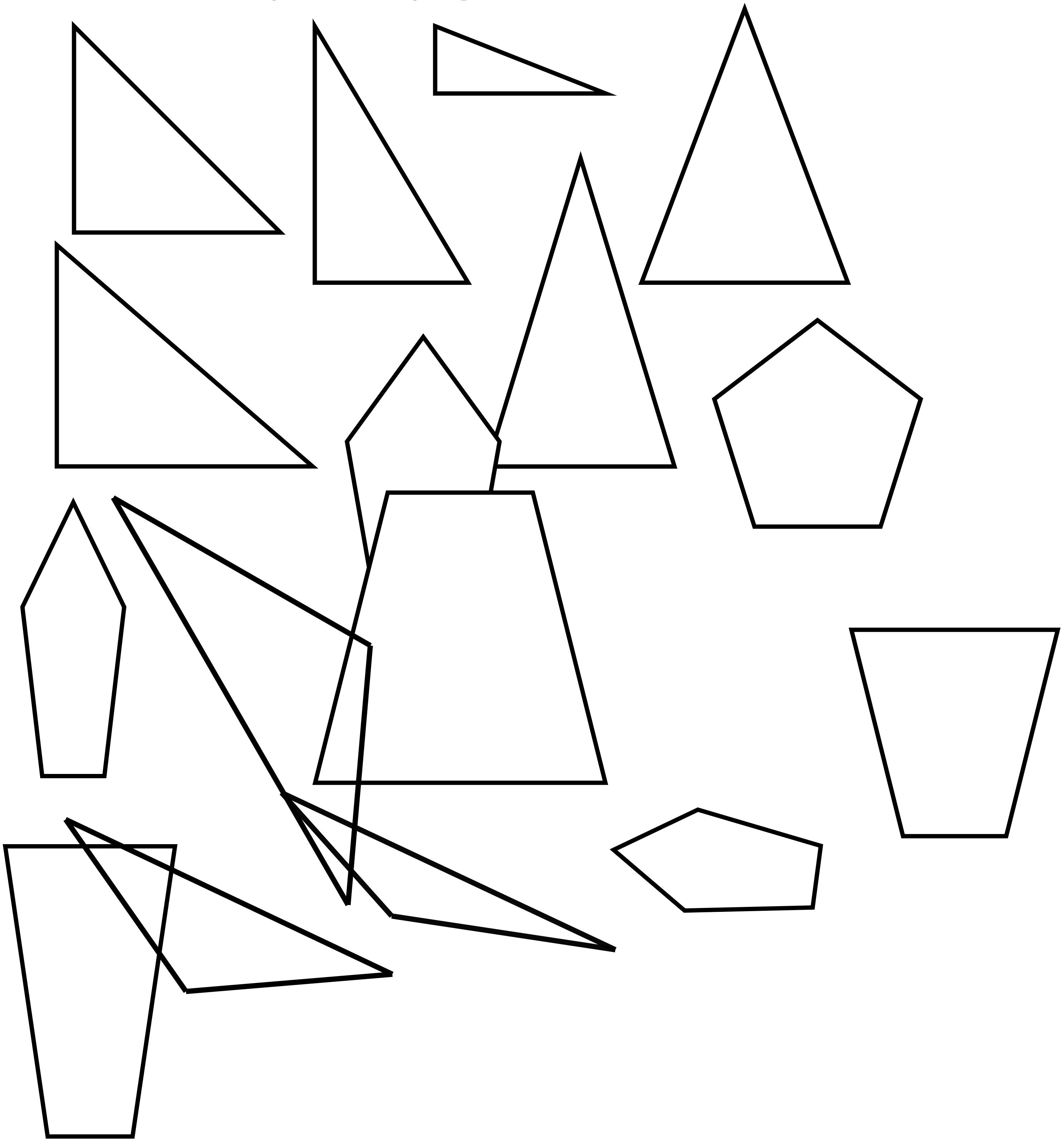


MAT D

Student Mats for Matching the Congruent Shapes. Match a shape from those that were cut out to one of these figures to find the congruent shape.



Worksheet 1A-2: Congruent Matching Shapes



Curriculum Unit Title: Where is Congruency in Your World?

Author: Doriel Moorman

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Two and three-dimensional objects can be described, classified, and analyzed by their attributes.

An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape.

ESSENTIAL QUESTION(S) for the UNIT

How does what we measure affect how we measure? How and why do we compare, contrast, and classify objects? How can transformations be described?

CONCEPT A	CONCEPT B	CONCEPT C
Appropriate tools to use for measuring	Describing, Classifying and Analyzing 2&3-dimensional objects	Transformations
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How does what we measure affect how we measure?	How do we describe, classify, compare, analyze, compare, and contrast angles and objects?	How can transformations be described?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
straight edge, compass, protractor, angle measure, side length	congruent, similar, angle, side, trilateral, triangle, quadrilateral, polygon, perpendicular, boundary, figure, rhomboid, rhombus, trapezia, trapezoid, parallelogram, obtuse, acute, right-angle	translation = slide, reflection = flip, rotation = turn

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Some of the terms used in Euclid's Elements are defined including proposition, theorem, proof, axiom, common notion, and postulate. The conditions for congruency of triangles is included Side-Side-Side, Angle-Side-Angle, side-Angle-Side. Activities involving computer access , interactive white board, and Geometer's Sketch Pad software are included.